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HAMMING DISTANCE LABELING OF CERTAIN GRAPHS

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Abstract:
Hamming distance is a metric used to compare two binary data strings of equal length, where binary string is a sequence of bytes which is used to store data in computers. It isthe number of bit positions in which the two binary strings differ. The hamming distance between two binary strings of equal length m and n is denoted by hd(m, n). The concept of hamming distance labeling was introduced by us and it has been proved that some path related graphs are hamming distance labeled graphs. Hamming distance labelingis used for transmitting secret messages in cryptography. In this paper, the existence of hamming distance labeling of twig, star, bistar and m- corona of path graph are verified and their hamming distance number are obtained.

Keywords: Hamming Distance, Hamming Distance Labeling, Hamming distancenumber, Path related graph.

1 Introduction

Let G be a simple, connected, undirected graph with vertex set V(G) and edge set E(G).Over the last few years, many graph labeling techniques were introduced. A detailed survey of such techniques was given by (Gallian, 2019). Adding to that we have introduced a new labeling called hamming distance labeling and is defined as follows: A function $ff: V \to N \cup \{0\}$ is said to be hamming distance labeling if there exist an induced function $ff^* : E \to \{1,2, \dots , n\}$ such that for every $uv \in E$,
$ff^*(uv) = hd([ff(u)]_2, [ff(v)]_2)$ satisfies the following conditions:
(i) for every vertex $v \epsilon\epsilon V$, the set of all edges incident with v receive distinct labels.
(ii) for every edge $e \epsilon\epsilon uv$, the adjacent vertices u and v receive distinct labels.
The hamming distance number of a graph G is the least positive integer n such that $2^n - 1 \geq k$, where $k = \max\{ff(v)/v \in V\}$ and is denoted by η_{hd}(G). Here the notation $[x]_2$ denotes the binary number of x. In this paper, we have shown that the twig graph, star graph, bistar graph and m-corona of path graph are hamming distance labeled graphs and obtained their hamming distance numbers. Here a Twig graph TW(P_m), $m \geq 2$ is a graph obtained from a path graph P_m by attaching exactly two pendant vertices to each internal vertices of the path graph (Durai Baskar & Manivannan, 2017, pp.55). where a path graph P_m of length m has m+1 vertices and m edges. A star graph S_m, $m \geq 2$ is the complete bipartite graph of the form $K_{1,-1}$ with m vertices (Esakkiammal, Thirusangu & Seethalakshmi, 2017, pp.94). The graph constructed by joining the apex vertices of two stars $K_{1,r}$ and $K_{1,s}$ for $r \geq 1$ and $s \geq 1$

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with disjoint vertex sets is known as the bistar $B_{r,s}$ graph(Ganesh & Revathi, 2018, pp.408) and the m-corona of Path graph P_m is obtained by joining the apex vertex of the star graph $K_{1,r}$ to each of the m+1 vertices of the Path graph P_m(Nikitha Prakash & Supriya Rajendran, 2018, pp.1897)

2 Main Results

Theorem 2.1. The Twig graph (P_m) is a hamming distance labeled graph and thehamming distance number is $\eta_{hd}(TTT(P_m)) = 4$, for any $m \geq 2$.

Proof: Let us consider the twig graph (P_m) with vertex set $V = \{\{v_i / 0 \leq i \leq m\} \cup \{v'_i, v''_i / 1 \leq i \leq m-1\}\}$ and edge set $E = \{\{v_i v_{i+1} / 0 \leq i \leq m-1\} \cup \{v_i v'_i / 1 \leq i \leq m-1\} \cup \{v_i v''_i / 1 \leq i \leq m-1\}\}$. Define a function $f: V \to N \cup \{0\}$ to label the vertices of $TTT(P_m)$ in such a way that $f(u) \neq f(v)$ for any two adjacent vertices and the procedure for labeling is given in the algorithm.

Procedure: Vertex labeling of (P_m), $m \geq 2$

Input: Vertices of (P_m) graph $V \to \{\{v_i / 0 \leq i \leq m\} \cup \{v'_i, v''_i / 1 \leq i \leq m-1\}\}$

$v_0 \leftarrow 0;$

$$v_m \leftarrow \begin{cases} 1, & for\ i \equiv 1 (mod 4) \\ 14, & for\ i \equiv 2 (mod 4) \\ 6, & for\ i \equiv 3 (mod 4) \\ 9, & for\ i \equiv 0 (mod 4) \end{cases}$$

for $i = 1$ to $m-1$ do

$$v_i \leftarrow \begin{cases} 1, & for\ i \equiv 1 (mod 4) \\ 14, & for\ i \equiv 2 (mod 4) \\ 6, & for\ i \equiv 3 (mod 4) \\ 9, & for\ i \equiv 0 (mod 4) \end{cases}$$

$$v'_i \leftarrow \begin{cases} 2, & for\ i \equiv 1 (mod 4) \\ 2, & for\ i \equiv 2 (mod 4) \\ 0, & for\ i \equiv 3 (mod 4) \\ 0, & for\ i \equiv 0 (mod 4) \end{cases}$$

$$v''_i \leftarrow \begin{cases} 6, & for\ i \equiv 1 (mod 4) \\ 0, & for\ i \equiv 2 (mod 4) \\ 1, & for\ i \equiv 3 (mod 4) \\ 2, & for\ i \equiv 0 (mod 4) \end{cases}$$

end for

end procedure

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Output: The labeled vertices of twig graph (P_m).The induced edge labels are as follows:

$f^*(v_0 v_1) = hd([f(v_0)]_2, [f(v_1)]_2) = hd(00000000 , 00000001) = 1.$

For $1 \leq i \leq m - 1$

$f^*(v_i v_i^{'}) = hd([f(v_i)]_2, [f(v_i^{'})]_2) = 2; f^*(v_i v_i^{''}) = hd([f(v_i)]_2, [f(v_i^{''})]_2) = 3;$

Case (i): if $\equiv 1(mod 2)$, $f^*(v_i v_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) =$

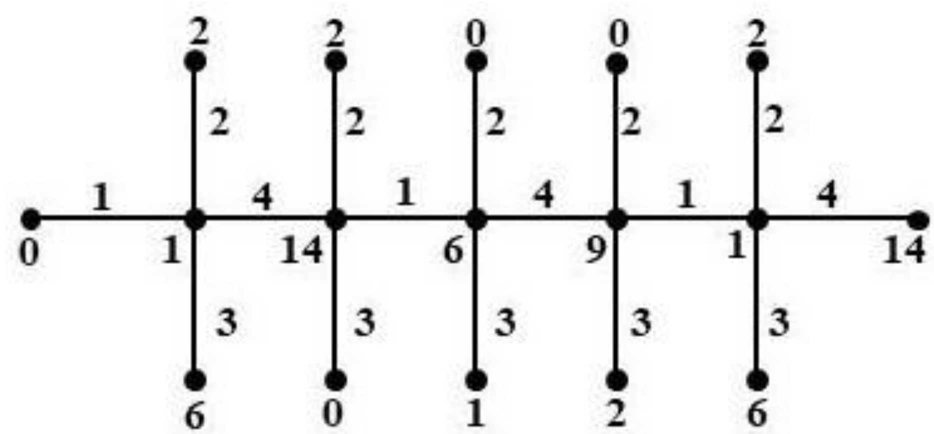
Case (i): if $\equiv 1(mod 2)$, $f^*(v_i v_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) =$

Case (ii): if $i \equiv 0(mod 2)$, $f^*(v_i v_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = 1.$

From all the above cases, all the adjacent edges receive distinct labels. Hence the twig graph $TTT(P_m)$ admits hamming distance labeling and the hamming distance number is $\eta_h(TTT(P_m)) = 4$, where $m \geq 2$.

Figure 1.

Hamming distance labeled (P_7) graph.



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Theorem 2.2. The Star graph S_m is a hamming distance labeled graph and the hamming distance number is $\eta_h(S_m) = m - 1$, for any $m \geq 2$.

Proof: Let us consider the star graph S_m with vertex set $V = \{v_i / 0 \leq i \leq m - 1\}$ and edge set $E = \{v_0 v_i / 1 \leq i \leq m - 1\}$. Define a function $f: V \to N \cup \{0\}$ to label the vertices of S_m in such a way that $(u) \neq f(v)$ for any two adjacent vertices and the procedure for labeling is given in the following algorithm.

Procedure: Vertex labeling of S_m, $m \geq 2$

Input: Vertices of S_m graph, $V \to v_i / 0 \leq i \leq m - 1\}$.

$v_0 \leftarrow 0$;

for $i = 1$ to $m - 1$ do

$v_i \leftarrow 2^i - 1$;

end for

end procedure

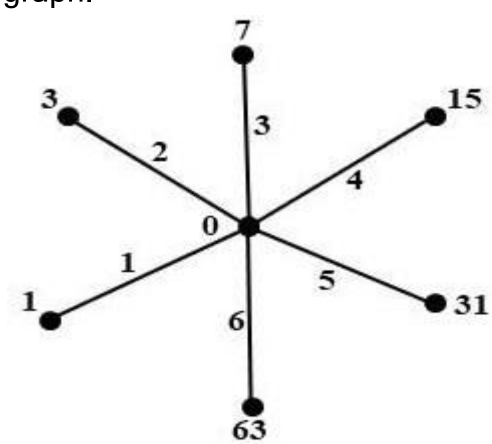
Output: The labeled vertices of star graph .The induced edge labels are as follows:

For $1 \leq i \leq m - 1$, $f^*(v_0 v_i) = hd([f(v_0)]_2, [f(v_i)]_2) = i$.

Here all the adjacent edges receive distinct labels. Hence the star graph S_m admits hamming distance labeling and the hamming distance number is $\eta_h(S_m) = m - 1$ for any $m \geq 2$.

Figure 2.

Hamming distance labeled S_7 graph.



Theorem 2.3. The Bistar graph $B_{r,s}$ is a hamming distance labeled graph and the hamming distance number is $\eta_{hd}(B_{r,s}) = \begin{cases} r + 1, & for\ r \geq s \\ s + 1, & for\ r < s \end{cases}$, for any $r \geq 1$ and $s \geq 1$.

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Proof: Let us consider the bistar graph $B_{r,}, r \geq 1$ and $s \geq 1$ with vertex set $V = \{\{u_i / 0 \leq i \leq r\} \cup \{v_i / 0 \leq j \leq s\}\}$ and edge set $E = \{\{u_0 u_i / 1 \leq i \leq r\} \cup \{u_0 v_0\} \cup \{v_o / 1 \leq j \leq s\}\}$. Define a function $f: V \rightarrow N \cup \{0\}$ to label the vertices of $B_{r,}$ in such a way that $(u) \neq f(v)$ for any two adjacent vertices and the procedure for labeling is given in the following algorithm.

Procedure: Vertex labeling of $B_{r,}$, $r \geq 1, s \geq 1$.

Input: Vertices of $B_{r,}$ graph,$V \rightarrow \{\{u_i / 0 \leq i \leq r\} \cup \{v_i / 0 \leq j \leq s\}\}$

for $i = 1$ to r do

$u_i \leftarrow 2^{(i+1)} - 1;$

end for

for j = 1 to s do

$v_j \leftarrow 2^{(j+1)} - 2;$

end for

end procedure

Output: The labeled vertices of Bistar graph $B_{r,}$ .The induced edge labels are as follows:

$f^*(u_0 v_0) = hd([f(u_0)]_2, [f(v_0)]_2),\ = hd(00000000\ , 00000001) = 1.$

For $1 \leq i \leq r$,$f^*(u_0 u_i) = hd([f(u_0)]_2, [f(u_i)]_2) =$ i+1.

For $1 \leq j \leq s$, $f^*(v_0 v_j) = hd([f(v_0)]_2, [f(v_j)]_2) =$ j+1.

Here all the adjacent edges receive distinct labels. Hence the bistar graph $B_{r,}$ admits hamming distance labeling and the hamming distance number is

$$\eta_{hd}(B_{r,}) = \begin{cases} r+1, & for\ r \geq s \\ s+1, & for\ r < s \end{cases}$$ for any $r \geq 1$ and $s \geq 1$.

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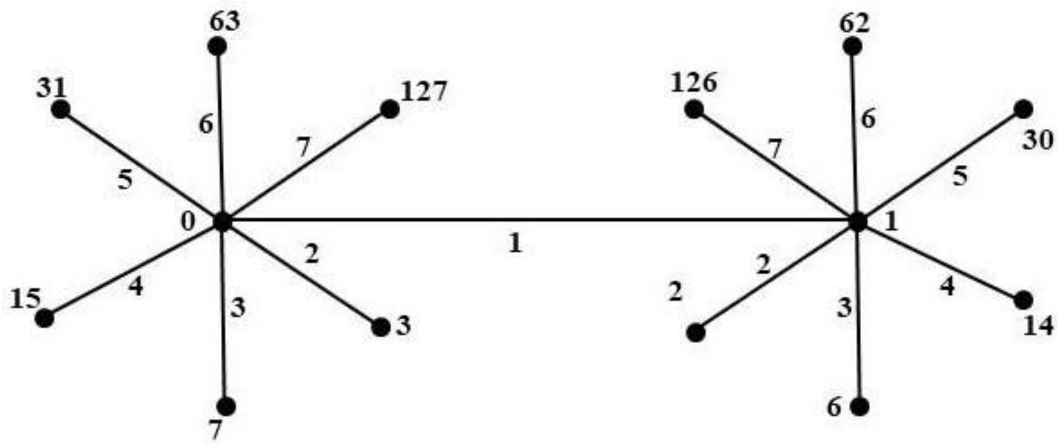
Figure 3.

Hamming distance labeled $B_{6,6}$ graph.



Theorem 2.4.The m-corona of path graph P_m is a hamming distance labeled graph and the hamming distance number is $\eta_h(m - corona\ of\ P_m) = 2^{r+2} - 1$.

Proof: Let us consider the m-corona of path graph P_m with vertex set $V = \{\{v_i \ /0 \le i \le m\} \cup \{v_{ij} /0 \le i \le m, 1 \le j \le r\}\}$ and edge set $E = \{\{v_i v_{i+1} \ / 0 \le i \le m - 1\} \cup \{v_i v_j \ / 0 \le i \le m, 1 \le j \le r\}\}$. Define a function $f: V \rightarrow N \cup \{0\}$ to label the vertices of $m - corona\ of\ P_m$ in such a way that $f(u) \neq f(v)$ for any two adjacent vertices and the procedure for labeling is given in the algorithm.

Procedure: Vertex labeling of $m - corona\ of\ path\ P_m,\ m \ge 1$

Input: Vertices of m -corona of path P_m graph,

$V = \{\{v_i \ /0 \le i \le m\} \cup \{v_{ij}/0 \le i \le m, 1 \le j \le r\}\}$

$v_0 \leftarrow 0$;
for $j = 1\ to\ r$ do
    $v_{0j} \leftarrow 2^{j+1} - 1$;
    for $i = 1\ to\ m$ do
        if $i \equiv 1 (mod 4)$
            $v_i \leftarrow 1$; $v_{ij} \leftarrow 2^{j+2} - 2$;
        else if $i \equiv 2 (mod 4)$
            $v_i \leftarrow 2$; $v_{ij} \leftarrow 2^{j+2} - 3$;
        else if $i \equiv 3 (mod 4)$
            $v_i \leftarrow 0$; $v_{ij} \leftarrow 2^{j+2} - 1$;

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else
$v_i \leftarrow 3$; $v_{ij} \leftarrow 2^{j+2} - 4$;
end if
end if
end if
end for
end for
end procedure

Output: The labeled vertices of m- corona of path graph P_m.The induces edge labels are as follows:

$f^*(v_0v_1) = hd([f(v_0)]_2, [f(v_1)]_2) = hd(00000000, 00000001) = 1$.

For $1 \le j \le r$, $f^*(v_0v_{0j}) = hd([f(v_0)]_2, [f(v_{0j})]_2)$ = j+1.

For $1 \le i \le m - 1$ & $1 \le j \le r$, $f^*(v_i v_{ij}) = hd([f(v_i)]_2, [f(v_{ij})]_2)$ = j+2.

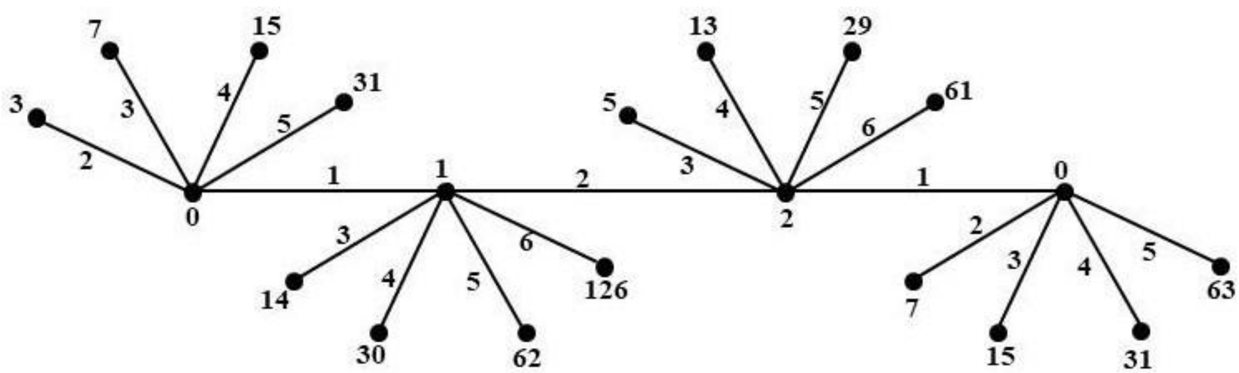
Case (i): if $i \equiv 1(mod2)$; $f^*(v_i v_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = 2$.

Case (ii): if $i \equiv 0(mod2)$; $f^*(v_i v_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = 1$.

from all the above cases, all the adjacent edges receive distinct labels. Hence the m-corona of path graph P_m admits hamming distance labeling and the hamming distance number is $\eta_h(m -$ corona of $P_m) = r + 2$.

Figure 4.

Hamming distance labeled $m - corona\ of f\ P_3\ graph.$



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References

- Durai Baskar, A & Manivannan, P.(2017).F-Heronian Mean Labeling of Graphs Obtained from Paths.International Journal of Pure and Applied mathematics, 117(5),55-62.
- Esakkiammal,E., Thirusangu,K., & Seethalakshmi,S. (2017). d-lucky labeling of arbitrary super subdivision of some graphs. International Journal of Pure and Applied Mathematics, 113(7), 93-101.
- Gallian,J.A.(2019). A Dynamic Survey of Graph Labeling.The Electronic Journal Of Combinatorics, #DS6.
- Ganesh,S.,& Revathi,R.(2018). Analysis of Some Bistar related MMD Graphs. International Journal of Pure and Applied Mathematics, 118(10), 407- 413.
- Nikitha Prakash & Supriya Rajedran.N.(2018).Neighbourhood prime labelling on some path related graphs. International Journal of Pure and Applied Mathematics, 118(20), 1893- 1901.